

# Ultrafast magnon-transistor at room temperature

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We study sequential tunneling of magnetic excitations in nonitinerant systems (either magnons or spinons) through triangular molecular magnets. It is known that the quantum state of such molecular magnets can be controlled by application of an electric- or a magnetic field. Here, we use this fact to control the flow of a spin current through the molecular magnet by electric- or magnetic means. This allows us to design a system that behaves as a magnon-transistor. We show how to combine three magnon-transistors to form a NAND-gate, and give several possible realizations of the latter, one of which could function at room temperature using transistors with a 11 ns switching time.

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## I. INTRODUCTION

In spintronic devices in insulating magnets, information about the logic state can be encoded in collective magnetic excitations, typically either spinons or magnons.<sup>1</sup> Due to the nature of these carriers, power dissipation in such nonitinerant devices is anticipated to be much lower<sup>2,3</sup> than in their electronic counterparts, as well as in spintronics devices in semiconductors.<sup>4</sup> As excess heating is a limiting factor in modern electronics, spintronics in insulating magnets is considered a contender to become the next computing paradigm.<sup>5–11</sup>

Since any classical algorithm can be implemented using a combination of transistors only, the design of this element in insulating magnets is a pivotal issue. Here, we theoretically show that it is possible to make a transistor in which the logic state is encoded in purely magnetic excitations, and whose operation can be controlled by either a magnetic- or electric field. In our transistor, triangular molecular magnets<sup>12–16</sup> take the role of gate, and we model the source and drain by spin reservoirs. We show that our transistor, which could operate at high clock speed at room temperature, can be used to implement the NAND-gate, one of the two existing universal gates for classical computation.

In our system, transport of magnetization occurs primarily by sequential tunneling of magnons (for FM reservoirs) or spinons (for AF reservoirs) through the molecular magnet. We will show how it is possible to suppress or increase this sequential tunneling (and thereby switch between the insulating and conducting state of the transistor) by changing the internal state of the molecule through external fields, either electric or magnetic. Molecular magnets are especially suitable due to their chemical variety and controllability, as well as their relatively large size, which makes control of the state easier. For similar reasons, they have been proposed as good building blocks for novel spin-polarized<sup>17</sup>, as well as quantum computing devices.<sup>18–20</sup>

This work is organized as follows: In Sec. II we introduce in more detail the previously mentioned system in which we will create our transistor. In Sec. III we cal-

culate the tunneling rates of magnons/spinons through a triangular molecular magnet, and calculate the spin current through the molecular magnet from these rates. In Sec. IV we show how controlling the state of a molecular magnet by electric- or magnetic fields allows us to design a transistor for either magnons or spinons. In Sec. V we focus on possible implementations of our transistor. Finally, we discuss certain constraints on our results in Sec. VI.

## II. SYSTEM

The system we employ consists of a triangular molecular magnet, such as  $\{\text{Cu}_3\}$  (see Ref. 21), with initially two vertices weakly exchange-coupled to identical spin reservoirs, see Fig. 1(a)-(b). We will consider both one-dimensional (1D) and two-dimensional (2D) FM spin reservoirs as well as 1D AF spin reservoirs.

Both spin reservoirs, the molecular magnet, and the weak coupling between the subsystems are initially described by the isotropic Heisenberg Hamiltonian with Dzyaloshinskii-Moriya (DM) interaction

$$H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j). \quad (1)$$

The exchange interaction  $J_{ij}$  and DM vector  $\mathbf{D}_{ij} = D_{ij} \hat{\mathbf{e}}_z$  are assumed to be constant for each subsystem, with values given in Fig. 1(b). We will assume that the  $J_i$ 's set the smallest energy scale in the system, so that we can analyze tunneling processes using perturbation theory. In our model, a finite spin current is induced by application of a magnetic field  $\Delta B$  to the left spin reservoir, which creates a non-equilibrium distribution of magnetic excitations. In reality, due to the finite lifetime of the magnetic excitations, a steady state spin current has to be generated using *e.g.* an AC magnet field difference, a static temperature difference, or spin pumping.

When  $D_M \ll J_M$  [see Fig. 1(b)], the low-energy subspace of the triangular magnetic molecule consists of a quadruplet with total spin-1/2, and the eigenstates of the

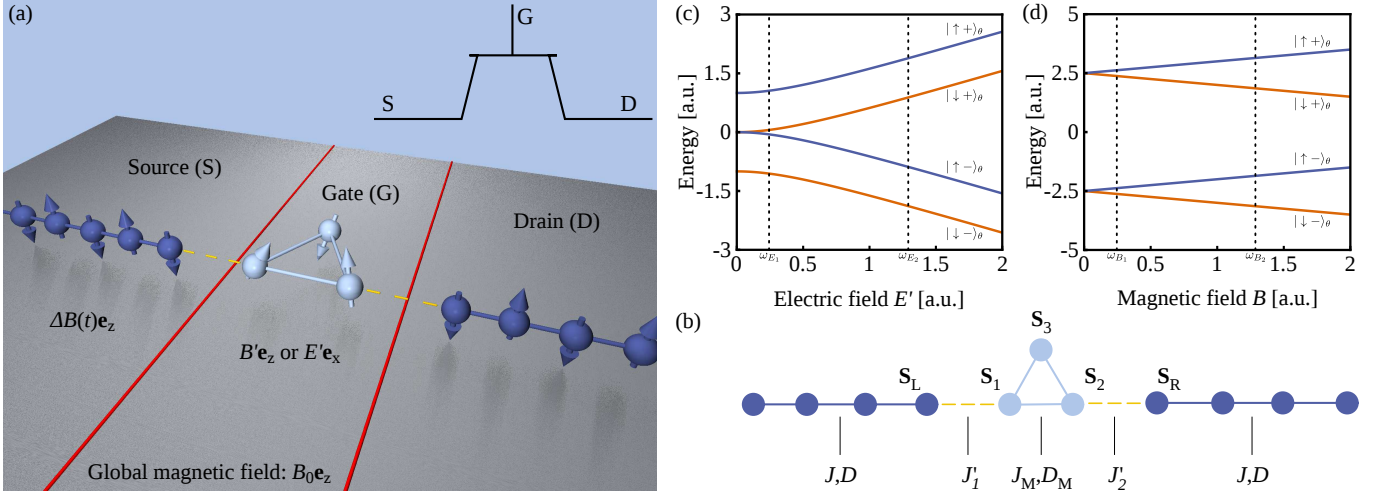


FIG. 1. (a)-(b) Pictorial representation of a single purely magnetic spin transistor, including Heisenberg parameters of the subsystems and the different magnetic- and electric fields. The field  $B_0$  is applied to both spin reservoirs (here shown as 1D spin chains) and the molecular magnet. (c)-(d) Excitation spectrum corresponding to Eq. (2), which can be rewritten in diagonal form as  $H_M = g_M \mu_B B S_\theta^z + \sqrt{(D_M/2)^2 + (dE')^2} C_\theta^z$ , where the subscript denotes an operator in the basis given by  $|\uparrow, 1\rangle_\theta = \sin\theta |\uparrow, 1\rangle + \cos\theta |\uparrow, -1\rangle$ ,  $|\uparrow, -1\rangle = -\cos\theta |\uparrow, 1\rangle + \sin\theta |\uparrow, -1\rangle$ , and  $|\downarrow, +\rangle_\theta, |\downarrow, -\rangle_\theta$  the same but with the spins flipped. We defined  $\tan\theta = [\sqrt{(D_M/2)^2 + (dE')^2} + D_M/2]/dE'$ . We have absorbed the spin-orbit interaction in the definition of the basis by performing a spin-dependent rotation in chirality space. In (c) we choose  $g_M \mu_B B = D_M = 10^{-3}$ , in (d) we put  $D_M = 5 \cdot 10^{-3}$ .

Hamiltonian can be labeled as  $|m_S, m_C\rangle$ . These states are eigenstates of the  $z$  projection of two mutually commuting effective spin-1/2 degrees of freedom: the total spin operator  $\mathbf{S}$  (with eigenvalues  $m_S = \pm 1/2$ ) and the chirality operator  $\mathbf{C}$  (with eigenvalues  $m_C = \pm 1$ ). An in plane electric field  $\mathbf{E} = E_x \hat{\mathbf{e}}_x + E_y \hat{\mathbf{e}}_y$  couples to the chirality through the spin-electric effect,<sup>22,23</sup> the coupling of a magnetic field  $\mathbf{B} = B \hat{\mathbf{e}}_z$  to the total spin is described by the Zeeman interaction. The Hamiltonian for the low-energy subspace is

$$H_M = g_M \mu_B B S^z + d \mathbf{E}' \cdot \mathbf{C}^\parallel + D_M S^z C^z. \quad (2)$$

Here,  $B = B' + B_0$ ,  $d$  is the effective dipole moment of the molecule,  $E'$  is the rotated electric field,<sup>22</sup> and  $\mathbf{C}^\parallel = C^x \hat{\mathbf{e}}_x + C^y \hat{\mathbf{e}}_y$ . The  $g$ -factor of the molecular magnet is denoted  $g_M$ . Diagonalization of Eq. (2) for  $\mathbf{E}' = E' \hat{\mathbf{e}}_x$  gives the spectrum depicted in Fig. 1(c)-(d).

Tunneling of magnetic excitations between the left-/right spin reservoir and the molecular magnet is described by the Heisenberg exchange interaction between  $\mathbf{S}_L$  and  $\mathbf{S}_1$  respectively  $\mathbf{S}_R$  and  $\mathbf{S}_2$ . By evaluating the relevant matrix elements of the Hamiltonian in the low-energy subspace of the molecular magnet, we find the respective effective Hamiltonians

$$\begin{aligned} H_{LM} &= \frac{J'_1}{3} [1 - 2 \sin(2\theta) C_\theta^z + 2 \cos(2\theta) C_\theta^x] \mathbf{S}_L \cdot \mathbf{S}_\theta, \\ H_{MR} &= \frac{J'_2}{3} [1 + \sin(2\theta) C_\theta^z - \cos(2\theta) C_\theta^x + \sqrt{3} C_\theta^y] \mathbf{S}_\theta \cdot \mathbf{S}_R. \end{aligned} \quad (3)$$

We interpret Eqs. (3) as effectively describing tunneling of spin excitations from the spin reservoir onto the total

spin of the molecular magnet and *vice versa*, with a tunneling strength that depends on the chirality state of the molecule.

We will also consider the case in which the third vertex of the molecule, described by the spin  $\mathbf{S}_3$ , is coupled to a single spin  $\mathbf{S}_C$  in a separate reservoir. Within the low-energy subspace of the molecule, we find the Hamiltonian

$$H_{CM} = \frac{J'_3}{3} [1 + \sin(2\theta) C_\theta^z - \cos(2\theta) C_\theta^x - \sqrt{3} C_\theta^y] \mathbf{S}_\theta \cdot \mathbf{S}_C. \quad (4)$$

Furthermore, in the case where a single spin  $\mathbf{S}_{L/R}$  is exchange-coupled to all three spins of the molecular magnet with equal strength  $J'$ , the effective Hamiltonian is simply  $J' \mathbf{S}_{L/R} \cdot \mathbf{S}_\theta$ . In this case, the tunneling of magnetic excitations no longer depends on the chirality of the molecule.

The first thing we see from Eqs. (3)-(4) is that a static equilibrium magnetization  $S \hat{\mathbf{e}}_z$  of a reservoir acts as a constant perturbation on the state of the molecular magnet through the relevant exchange interaction between reservoir and molecule. We will show that it is possible to make this effect trivial, or even beneficial to our purposes, in all cases under consideration. Additional dynamics of the systems is due to the behavior of magnetic excitations which exist on top of the equilibrium magnetization. We will study this dynamics next.

### III. TRANSITION RATES AND SPIN CURRENT

To determine the spin current through the molecular magnet, we use a master-equation approach. We assume that energy is conserved in all tunneling processes, and ignore higher order effects. Transition rates from initial state  $|i\rangle$  to final state  $|f\rangle$  of the system due to the tunneling processes described by Eqs. (3) are described respectively by  $R_{if}^L$  and  $R_{if}^R$ . Using the golden rule, we can calculate  $R_{if}^{L,R}$  to second order in  $J'_{1,2}$  as

$$R_{if}^L = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} d\tau \langle i | H'_{LM}(\tau) | f \rangle \langle f | H'_{LM}(0) | i \rangle. \quad (5)$$

The same expression holds for  $R_{if}^R$  with  $H'_{LM}$  replaced by  $H'_{MR}$ . The apostrophe denotes an operator in the interaction representation. The nontrivial part of the problem reduces then to finding correlation functions such as  $\langle S_{L/R}^+(\tau) S_{L/R}^-(0) \rangle$  in the spin reservoirs. In the next two sections we will find the relevant expressions for both FM and AF spin reservoirs.

Calculation of the spin current requires both the transition rates as well as the probabilities  $P_i$  that the molecule is in the state  $|i\rangle$ . We define the vector  $\mathbf{P} = (P_{\uparrow+}, P_{\uparrow-}, P_{\downarrow+}, P_{\downarrow-})$ . The time evolution of the occupation probability vector  $\mathbf{P}$  is then given by  $d\mathbf{P}/dt = \hat{R}\mathbf{P}$ , where  $\hat{R}$  is the 4x4 matrix that contains the appropriate transition rates. The steady state probabilities are contained in the kernel of  $\hat{R}$ , normalized such that  $\sum_i P_i = 1$ . Hence,  $\mathbf{P}$  is uniquely determined by the transition rates. The spin current  $I_S$  is then defined as the net rate with which excitations leave the left reservoir

$$I_S = (R_{\downarrow+\uparrow+}^L + R_{\downarrow+\uparrow-}^L) P_{\downarrow+} + (R_{\downarrow-\uparrow+}^L + R_{\downarrow-\uparrow-}^L) P_{\downarrow-} - (R_{\uparrow+\downarrow+}^L + R_{\uparrow+\downarrow-}^L) P_{\uparrow+} - (R_{\uparrow-\downarrow+}^L + R_{\uparrow-\downarrow-}^L) P_{\uparrow-}. \quad (6)$$

Next, we will calculate the relevant transition rates for the different types of reservoirs.

#### A. FM reservoirs

We first consider the simplest case of 1D FM reservoirs in the absence of DM interaction, *i.e.* with  $D = 0$ . Using the Holstein-Primakoff transformation, we can map a 1D FM system with  $S \gg 1/2$  on a system of non-interacting bosonic particles (magnons) with dispersion  $\hbar\omega_q = 4|J|S \sin^2(qa/2) + g_R \mu_B B_0$ , where  $g_R$  is the  $g$ -factor of the reservoir. This allows us to find the required correlation functions by rewriting  $\langle S_{L/R}^+(\tau) S_{L/R}^-(0) \rangle$  etc. in terms of the bosonic operators  $a_q^\dagger, a_q$ . We find the

rates

$$R_{\uparrow+\downarrow+}^L = \left( \frac{J'_1 \xi_+^L}{3\hbar} \right)^2 \frac{S}{2\pi} K_{\text{FM}}(\omega_B - \omega_{\Delta B}), \quad (7)$$

$$R_{\uparrow+\downarrow-}^L = \left( \frac{J'_1 \eta_+^L}{3\hbar} \right)^2 \frac{S}{2\pi} K_{\text{FM}}(\omega_B + \omega_E - \omega_{\Delta B}).$$

The energy scales are given by  $\hbar\omega_B = g_M \mu_B B$ ,  $\hbar\omega_{\Delta B} = g_R \mu_B \Delta B$ , and  $\hbar\omega_E = 2\sqrt{(D_M/2)^2 + (dE')^2}$ . Also,  $\xi_+^L = 1 - 2\sin(2\theta)$  and  $\eta_+^L = 2\cos(2\theta)$ . Furthermore,  $K_{\text{FM}}(\omega) = \rho_{1D}(\omega_q) [1 + n_B(\omega_q)]|_{\omega_q=\omega}$ , where  $\rho_{1D}(\omega_q) = a |\partial\omega_q/\partial q|^{-1}$  is proportional to the density of states (DOS) in the spin chain, and  $n_B(\omega_q)$  is the Bose-Einstein distribution of the magnons. To get the other rates, we use that the only effect of inverting the total spin (in the initial and final state simultaneously) is to replace  $1 + n_B(\omega_q) \rightarrow n_B(\omega_q)$  and  $\omega_E \rightarrow -\omega_E$ , and inverting both the chiralities changes  $\omega_E \rightarrow -\omega_E$ ,  $\xi_+^L \rightarrow \xi_-^L = 1 + 2\sin(2\theta)$ , and  $\eta_+^L \rightarrow \eta_-^L = \eta_+^L$ . To obtain the rates with respect to the right spin chain, we put  $\omega_{\Delta B} = 0$  in Eqs. (7). Furthermore, we replace  $\xi_\pm^L \rightarrow \xi_\pm^R = 1 \pm \sin(2\theta)$ ,  $\eta_\pm^L \rightarrow \eta_\pm^R = |\cos(2\theta) + i\sqrt{3}|$ , and  $J'_1 \rightarrow J'_2$ . Due to energy conservation, processes that do not flip the total spin can only occur for  $\omega_E = 0$ . Since we will always consider finite  $\omega_E$ , we can put these rates to zero.

To obtain the rates for a system with a 2D FM reservoir, we simply replace  $\rho_{1D}(\omega_q)$  by the 2D DOS, which for small  $|\mathbf{q}|$  is given by  $\rho_{2D}(\omega_q) = \hbar/(4S|J|)$ .

#### B. AF reservoirs

Next, we will derive the transition rates for hopping of spinons between semi-infinite AF spin-1/2 chains and a triangular molecular magnet. In order to do so, we start by giving a description of the spin chains in terms of Luttinger liquid theory, which turns out to be a convenient framework for our purpose. For concreteness, we focus on the description of the left spin chain. Eq. (1) can be mapped on the anisotropic Heisenberg Hamiltonian with anisotropy  $\Delta = J/\sqrt{J^2 + D^2}$  by performing a position-dependent rotation in spin space. After performing a Jordan-Wigner transformation, taking the continuum limit of the resulting fermionic Hamiltonian, and subsequent bosonization, the resulting Hamiltonian describing the left spin chain [for which  $x \in (-\infty, 0]$ ] becomes

$$H_L = \frac{\hbar}{2\pi} \int_{-\infty}^0 dx \left[ uK (\partial_x \theta(x))^2 + \frac{u}{K} (\partial_x \phi(x))^2 \right]. \quad (8)$$

The bosonic density field  $\phi(x)$  and its conjugate momentum field  $\theta(x)$  satisfy  $[\phi(x), \partial_{x'} \theta(x')] = i\pi \delta(x - x')$ . The sound velocity  $u$  of the bosonic excitations as well as the interaction parameter  $K$  can be determined from the parameters  $J$  and  $D$  of the spin chain using Bethe Ansatz results. At the isotropic point  $K = 1/2$ , and  $K = 1$

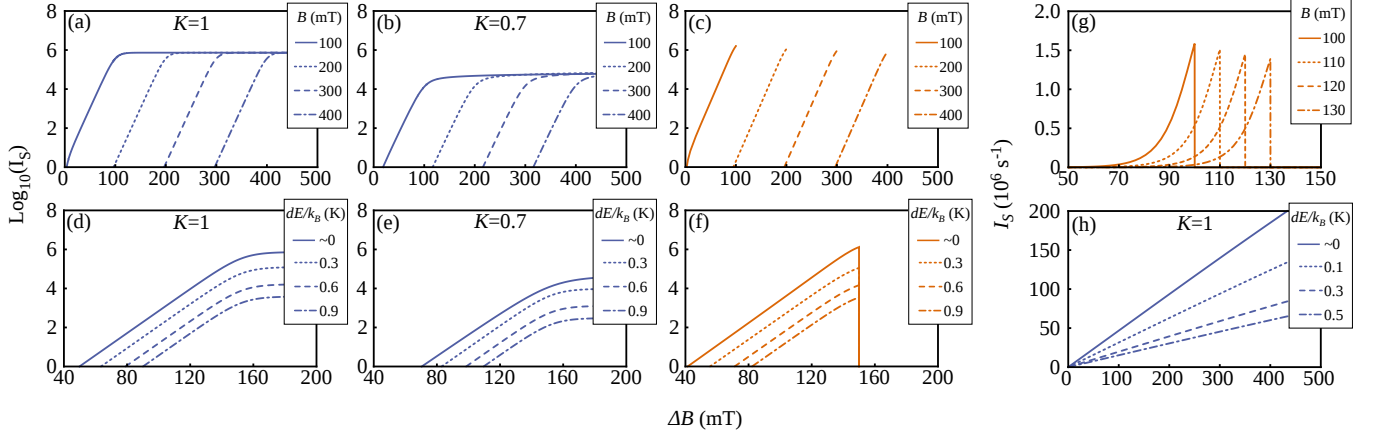


FIG. 2. (a)-(b)  $\log_{10}(I_S)$  versus  $\Delta B$  for different values of the magnetic field  $B$ , for 1D AF reservoirs with different Luttinger liquid parameters  $K$ . Due to the gapless nature of the spinons, we can always set  $B = B_0$  and  $B' = 0$  when considering AF reservoirs. (c) Idem for 1D FM reservoirs. Here,  $B_0 = 1 \mu\text{T}$  and  $B' \approx B$ . (d)-(e)  $\log_{10}(I_S)$  versus  $\Delta B$  for different values of  $dE'$ , for AF reservoirs with different Luttinger liquid parameters  $K$ . We choose  $D_M/k_B = 0.6 \text{ K}$  and  $B = 150 \text{ mT}$ . If we assume that the effective dipole moment lies between  $d = (10^{-4} - 1)eR$  (see Ref. 23), where  $R \approx 1 \text{ nm}$  is the bond length of the molecular magnet, then  $dE'/k_B = 0.1 \text{ K}$  corresponds to an electric field  $E' \sim (10^8 - 10^4) \text{ V m}^{-1}$ . (f) Idem for the FM system. Here,  $B_0 = 1 \mu\text{T}$  and  $B' = 150 \text{ mT}$ . (g) Illustration of the alternative switching mechanism for FM reservoirs. When the level splitting of the molecular magnet is smaller than the minimum energy of a magnon in the lead, the system is in the insulating phase. Again,  $B_0 = 1 \mu\text{T}$ . The plots (a)-(g) are for parameters  $J/k_B = 100 \text{ K}$ ,  $T = 10 \text{ mK}$ , and  $J'_1/k_B = J'_2/k_B = 0.1 \text{ K}$  (see text). For the FM plots,  $S = 1$ . (h)  $I_S$  versus  $\Delta B$  for different values of  $dE'$  for the AF system, at an experimentally accessible temperature. Parameters are  $J/k_B = 100 \text{ K}$ ,  $J'/k_B = 2 \text{ K}$ ,  $D_M/k_B = 0.3 \text{ K}$ ,  $B = 75 \text{ mT}$ , and  $T = 1 \text{ K}$ .

describes the free model. To analyze the hopping between spin chain and molecular magnet, we will need the continuum form of the spin operators in the spin chain. These are given by

$$\begin{aligned} S^-(x) &= \frac{e^{i\theta(x)}}{\sqrt{2\pi a}} (-1)^x [\cos(2\phi(x) - 2k_F x) + 1], \\ S^z(x) &= -\frac{1}{\pi} \partial_x \phi(x) + \frac{1}{\pi a} \cos[2\phi(x) - 2k_F x]. \end{aligned} \quad (9)$$

Here,  $a$  is the lattice spacing of the spin chain, and  $k_F = \pi/2a + g_R \mu_B B_0 / \hbar v$  is the Fermi wave vector. For simplicity, we assume that  $B_0 = 0$  in our derivation. Since the spin chain is semi-infinite (with its last spin at  $x = -a$ ), we require that the wave function vanishes at the origin. This constrains the density field to a constant value at the origin, such that  $\cos \phi(0) = 0$ . To analyze the behavior of the spin fields at the origin, we introduce chiral fields  $\phi_{L/R}(x) = K\theta(x) \pm \phi(x)$ , which are related on the entire space by the constraint on the density field at the origin.<sup>24</sup> This allows us to map Eq. (8) on a quadratic Hamiltonian that only depends on  $\phi_L(x)$ . Performing a renormalization group (RG) analysis on the spin operators near the boundary then yields that  $S^z(x)$  is a marginal operator, and  $S^\pm(0)$  scales as  $1 - 1/(2K)$ , so that it is relevant for systems with finite DM interaction. Since the RG flow is stopped either by temperature or by the relevant energy scale  $E_M$  of the molecular magnet, this gives the constraint on the hopping  $J' [\max(k_B T, E_M)/J]^{-1+1/(2K)} \ll J$  for our sequential tunneling approach to be valid. We note that when

$\omega_B = 0$  and  $J_M, D_M \gg J, D$ , the entire system can be described as two spin chains connected by a single impurity spin. It is known that, at half-filling (*i.e.*  $B_0 = 0$ ), such a system 'heals' itself at low temperatures,<sup>24</sup> and the current at small  $\Delta B$  becomes that of the free system,  $I_S = (g\mu_B)^2 \Delta B / \hbar$ . In this regime, the current in the conducting state is strongly enhanced compared to that in the sequential tunneling regime.

In order to calculate the required transition rates, we need to calculate spin-spin correlation functions at the boundary. Since the density field is frozen out there, the sole relevant correlation function is that of the momentum field  $\theta(t) \equiv \theta(0, t)$ . At finite temperature  $T$ , it is given by

$$\langle [\theta(t) - \theta(0)]^2 \rangle = \frac{2}{K} \ln \left[ \left( \frac{i\hbar\omega_C}{\pi\theta_0} \right) \sinh \left( \frac{\pi\theta_0 [t - i\delta]}{\hbar} \right) \right]. \quad (10)$$

Here,  $\theta_0 = k_B T$ , and  $\omega_C$  is the UV-cutoff of the theory. For this model it is approximated as  $\omega_C \approx J/\hbar$ .  $\delta$  is a positive infinitesimal. The analysis of the right spin chains goes along the same lines, and we will refrain from repeating the steps here.

To calculate the transition rates for AF reservoirs we substitute Eqs. (9) in Eq. (5). Using the correlation function Eq. (10) and the fact that  $\phi(x)$  is constant at

the boundary then gives the rates

$$\begin{aligned} R_{\uparrow+\downarrow+}^L &= \left( \frac{J_1' \zeta_+^L}{3\hbar\omega_C} \right)^2 K_{\text{AF}}(\omega_B - \omega_{\Delta B}), \\ R_{\uparrow+\downarrow-}^L &= \left( \frac{J_1' \nu_+^L}{3\hbar\omega_C} \right)^2 K_{\text{AF}}(\omega_B + \omega_E - \omega_{\Delta B}). \end{aligned} \quad (11)$$

Here,  $\zeta_+^L = d_L[1 - 2\sin(2\theta)]/2$ ,  $\nu_+^L = d_L \cos(2\theta)$ , and  $d_L = \sqrt{2/\pi}$ . We have ignored a small  $k_F$ -dependent contribution to  $d_L$  here. The function  $K_{\text{AF}}(\omega)$  describes the influence of the spin chain on the transition rate and is given by

$$\begin{aligned} K_{\text{AF}}(\omega) &= \omega_C^2 \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} e^{-\frac{1}{2}[\theta(\tau) - \theta(0)]^2} \\ &= \omega_T \left( \frac{\omega_T}{\omega_C} \right)^{-2+\frac{1}{K}} e^{\pi\omega/\omega_T} \frac{|\Gamma(1/(2K) + i\omega/\omega_T)|^2}{\Gamma(1/K)}. \end{aligned} \quad (12)$$

Here,  $\omega_T = 2\pi\theta_0/\hbar$ . To get the other rates, we use that the only effect of inverting the total spin (in the initial and final state simultaneously) is to change the sign of  $\omega_B$  and  $\omega_{\Delta B}$ , and inverting both the chiralities changes  $\omega_E \rightarrow -\omega_E$ ,  $\zeta_+^L \rightarrow \zeta_-^L = d_L[1 + 2\sin(2\theta)]/2$ , and  $\nu_+^L \rightarrow \nu_-^L = \nu_+^L$ . To obtain the rates with the respect to the right spin chain, we put  $\omega_{\Delta B} = 0$  in Eqs. (11). Furthermore, we replace  $\zeta_{\pm}^L \rightarrow \zeta_{\pm}^R = d_R[1 \pm \sin(2\theta)]/2$ ,  $\nu_{\pm}^L \rightarrow \nu_{\pm}^R = d_R|\cos(2\theta) + i\sqrt{3}|/2$ , and  $J_1' \rightarrow J_2'$ . The constant  $d_R = d_L$ . For the AF reservoirs, as for the FM reservoirs, processes that do not flip the total spin can only occur for  $\omega_E = 0$ . Since we will always consider finite  $\omega_E$ , we can put these rates to zero.

At this point, we have derived the transition rates for both AF and FM reservoirs. As we have seen, the main difference in the resulting rates is the replacement of the bosonic DOS and distribution function by power law functions, typical for Luttinger liquid models. We note here that, at low energies, the bosonic character of the magnons in the FM system yields larger spin currents than the fermionic spinons, which is extremely beneficial for the application we have in mind here.

#### IV. TRANSISTOR BEHAVIOR

Next, we show how our setup can be used as a logic switch whose working is controlled by an external magnetic field. For the first example we assume that  $k_B T \ll \hbar\omega_{\Delta B}, \hbar\omega_B \ll D_M$ , so that we only need to take the states with chirality -1 into account. Referring back to Fig. 1(c), we see that spin transport through the molecule will be strongly suppressed for magnetic field differences such that  $\omega_{\Delta B} \ll \omega_B$ , since in this regime the vast majority of the excitations in the reservoirs lack the required energy to induce a spin-flip on the molecule; For  $\omega_{\Delta B} \lesssim \omega_B$ , transport increases rapidly with  $\omega_{\Delta B}$ . Hence, for magnetic field gradients  $\omega_{\Delta B} \approx \omega_{B_1}$ , our setup can

be switched between the insulating- (for  $\omega_B \approx \omega_{B_2}$ ) and conducting (for  $\omega_B \approx \omega_{B_2}$ ) state. This is shown in Fig. 2(a)-(c).

The system with FM reservoirs offer an additional possibility to switch between the insulating and conducting state: When  $\omega_{\Delta B} > \omega_B$ , the minimum energy of the magnons in the reservoir exceeds the level splitting of the molecular magnet. In this case, the system is also insulating (neglecting higher order processes). This behavior has been indicated in Fig. 2(g). The use of this mechanism to switch between insulating and conducting states requires smaller magnetic fields compared to the previously discussed mechanism.

Alternatively, the switching behavior can be controlled by an electric field. The mechanism is different from that for magnetic control, since an electric field does not increase the splitting between the two lowest states with opposite total spin. However, we note that when  $k_B T \ll \hbar\omega_E$ , transport occurs through transitions between states in the subspace spanned by  $|\uparrow, -1\rangle_\theta$  and  $|\downarrow, -1\rangle_\theta$ . Eqs. (3) show that when  $\theta \rightarrow \pi/4$ , *i.e.* when  $dE' \gg D_M$ , the molecular magnet and the right spin reservoir are effectively decoupled in this low-energy subspace. Fig. 2(d)-(f) shows this switching behavior as a function of applied electric field.

In Fig. 2 we assumed in plane Heisenberg exchange interaction between reservoirs and vertices for all AF reservoirs. This is reasonable, since the AF reservoirs which we consider are themselves anisotropic. For FM reservoirs we assumed isotropic coupling  $J_1'$  and  $J_2'$ . Additionally, we assumed that the third vertex of each molecular magnet is coupled to a separate reservoir by an Ising-like interaction with strength  $J_3' = J'$ . The reason behind this assumption is that in this way the sole effect of the equilibrium magnetization of the reservoirs on the Hamiltonian of the molecular magnets is to act as an approximate effective magnetic field  $J'S\hat{\mathbf{e}}_z$ . This effective field for the parameters in Fig. 2 (a)-(g) is on the order of 75 mT, and can take the role of  $B'$ , which reduces the required external magnetic field.

#### V. EXPERIMENTAL REALIZATIONS

A single magnetic dipole moving with constant velocity  $\mathbf{v}$  gives rise to a magnetic field  $\mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{q\mu_B}{r^3} [3(\hat{\mathbf{e}}_z \cdot \hat{\mathbf{e}}_r)\hat{\mathbf{e}}_r - \hat{\mathbf{e}}_z]$  as well as an electric field  $\mathbf{E}_{\text{dip}}(\mathbf{r}) = \mathbf{v} \times \mathbf{B}_{\text{dip}}(\mathbf{r})$  in the laboratory frame. Conceivably, it is therefore possible to use the setup depicted in Fig. 3(a) to measure the switching behavior of a collection of spinon-transistors due to the spin-electric coupling at temperatures of  $\sim 1$  K. This can be done by measuring the difference in voltage drop between points  $(0, 0, r)$  and  $(0, r, 0)$  in the insulating- [at  $dE' \approx D_M/3$ ] and conducting (at  $dE' \approx 0$ ) state. For the parameters in Fig. 2(h), and for  $\Delta B \approx 200$  mT, the difference in spinon current between the two states is  $\sim 3 \cdot 10^{10}$  spinons  $\text{s}^{-1}$ . For  $r = 1 \mu\text{m}$ , this leads to a difference in voltage drop of  $\sim 10^{-13}$

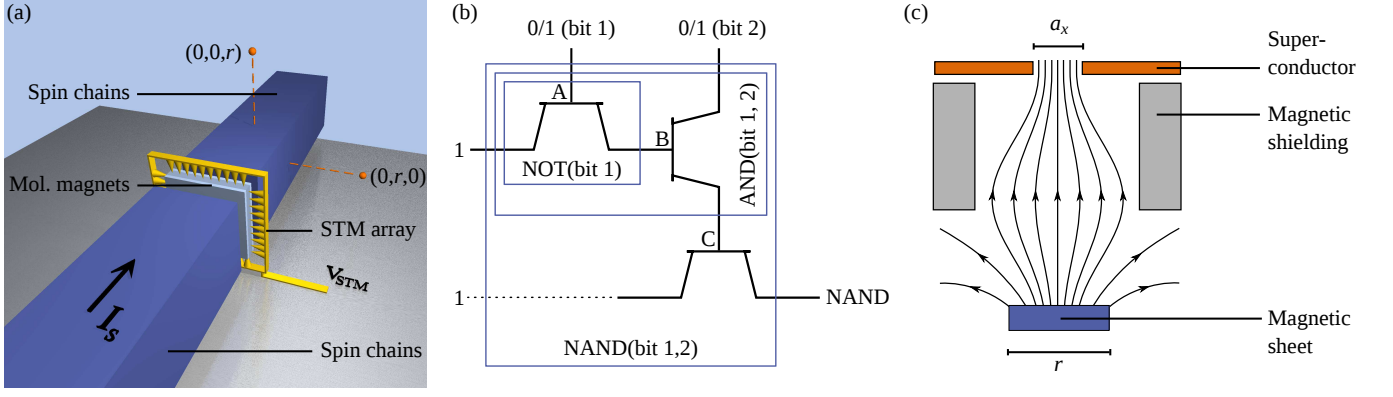


FIG. 3. (a) Proposed setup to measure the switching effect due to the spin-electric effect. The setup consists of a single-layer crystal of triangular magnets, of dimension  $0.2 \times 0.2 \mu\text{m}$ . The crystal contains  $\sim 4 \cdot 10^4$  molecules, given a lattice constant of 1 nm. The crystal is weakly exchange-coupled to a bulk collection of AF 1D spin chains, such as is realized in  $\text{SrCuO}_2$  (Ref. 25) or  $\text{Cs}_2\text{CoCl}_4$  (Ref. 26). Assuming the lattice constant is commensurate with that of the crystal, the setup contains  $\sim 4 \cdot 10^4$  parallel transistors. The  $\sim 10^3$  molecules at the edge can be accessed electrically by an array of STM tips. Using our previous estimate for  $d$  and the values from the text, fields between  $10^8 - 10^4 \text{ V m}^{-1}$  are required to switch between insulating and conducting state. (b) Combining three transistors into a single NAND-gate. (c) Proposed setup to enhance the magnetic field due to the accumulated magnons. The crystal of molecular magnets is placed directly above the hole in the superconductor. We denote by  $f$  the portion of the total flux directly on the magnet that can be enhanced into the area of the molecular magnets.

$V$ , which is within experimental reach.<sup>27</sup> The strength of the required switching field  $E'$  can be achieved near a STM-tip for molecules with reasonable  $d$ , see the caption of Fig. 3(a). This experiment would be interesting in its own right, since to our knowledge there have been no measurements of the spin-electric effect yet.

By combining three transistors as shown in Fig. 3(b), we can create a purely magnetic NAND-gate. The NAND-gate is a two-bit gate that gives a logical 0 as outcome if and only if both the input bits are 1, and yields a logical 1 otherwise. We will show how it is possible to implement a NAND-gate consisting of magnon-transistors using readily available materials at  $\sim 10 \text{ K}$ , and we will indicate the requirements for a working NAND-gate at room temperature. A finite non-equilibrium magnetization of a terminal (reservoir) encodes the logical state 1; the logical state 0 has only the equilibrium magnetization present. We propose to use the magnetic dipole field  $\mathbf{B}_{\text{mag}} = B_{\text{mag}}\hat{\mathbf{e}}_z$  due to the excess  $N$  accumulated magnons in the 1-state of the terminal as gate-field at the points A-C in Fig. 3(b). We use the setup in which each reservoir is coupled to all three vertices equally, with strength  $J'$ . We assume that one reservoir is coupled antiferromagnetically, and the other ferromagnetically. In this way, the effects of the equilibrium magnetization of the reservoirs on the state of the molecular magnet cancel each other. We use the fact that for  $\omega_{\Delta B} > \omega_{B_{\text{mag}}}$  the system is insulating, and for  $\omega_{\Delta B} < \omega_{B_{\text{mag}}}$  the system is conducting. Due to thermal fluctuations, the magnetic field  $\mathbf{B}_{\text{mag}}$  is not constant. These fluctuations limit the fidelity of our NAND-gate. We characterize the fluctuations by the standard deviation  $\sigma = \sqrt{\langle [\hat{n} - \langle \hat{n} \rangle]^2 \rangle}$  of the number of magnons  $n$  on the gate, and calculate  $\sigma$  using the equilibrium distribu-

tion of the magnons using the grand canonical ensemble. We consider first 2D FM reservoirs with spin  $S = 10$ ,  $J/k_B = 5 \text{ K}$ , and lattice spacing  $a = 1 \text{ nm}$  (the approximate values for YIG). For the source and drain, we consider a single layer sample of dimensions  $300 \text{ nm} \times 300 \text{ nm} \times 1 \text{ nm}$ . We put  $J'/k_B = 1 \text{ K}$ . As gate, we use a crystal of molecular magnets with  $D_M/k_B = 0.5 \text{ K}$ , and  $J_M/k_B \gg 10 \text{ K}$ . Assuming matching lattice constants, this setup contains 300 parallel single-molecule transistors. We estimate the field at the position of the molecular magnet [see Fig. 3(c)] due to the  $n$  magnons as  $|\mathbf{B}_{\text{mag}}| \approx \frac{1}{2}\mu_0 n g \mu_B f / (d a_x a_y)$ . To get rid of the magnetic field due to the equilibrium magnetization, we can add an additional FM with antiparallel equilibrium magnetization and no excess magnons on the opposite side of the molecule. Additionally, we use parameters  $g_M = g_R = 2$ ,  $f = 0.15$ ,  $B_0 = 200 \text{ mT}$ ,  $B' = 60 \text{ mT}$ , and  $\Delta B = 50 \text{ mT}$ . In the conducting state, we find a magnon current exceeding  $1.5 \cdot 10^{10} \text{ s}^{-1}$ , which amounts to a switching time of a transistor of  $\sim 300 \text{ ns}$ . In our model, the fidelity of a single-NAND-gate then exceeds 99.9%.

By using materials with an increased  $g$ -factor, we can create a NAND-gate that functions at room temperature, consisting of transistors with a  $\sim 11 \text{ ns}$  switching time. We use the same setup as in the previous paragraph, but with parameters  $S = 3$ ,  $J/k_B = 500 \text{ K}$ ,  $a = 1 \text{ nm}$  for the 2D FM reservoir;  $D_M/k_B = 2.5 \text{ K}$ , and  $J_M/k_B \gg 300 \text{ K}$  for the molecular magnet; and  $J'/k_B = 100 \text{ K}$ ,  $g_M = g_R = 20$ ,  $f = 0.15$ ,  $B_0 = 1 \text{ T}$  (easily achievable near the surface of a FM),  $B' = 100 \text{ mT}$ , and  $\Delta B = 50 \text{ mT}$ . We find a magnon current exceeding  $2.7 \cdot 10^{11} \text{ s}^{-1}$  in the conducting state, which corresponds to a 11 ns switching time. Reducing the magnon fluctuations on the gate can



further reduce the switching time. As before, the fidelity of a single NAND-gate exceeds 99.9%.

## VI. DISCUSSIONS

In this section, we will discuss several different requirements that have to be fulfilled for our perturbative calculations of the tunneling current through the molecular magnet to be valid. The first constraint on our calculations concerns the validity of our spin wave analysis of the FM spin chains; the number of magnons per site has to satisfy  $\langle a_i^\dagger a_i \rangle \ll 2S$ . In our calculations, the average number of magnons per site is typically 0.05-0.13, so that non-interaction spin wave theory is valid. The AF theory is valid for energies much smaller than the exchange interaction  $J$ .

We checked the validity of our sequential tunneling approach in a self-consistent manner. For the AF reservoirs, the criterion is simply that the tunneling current is much smaller than the current in the ballistic system, that is  $I_S(\Delta B) \ll g\mu_B\Delta B/h$ . For the FM reservoirs, we require that the broadening of the energy levels of the molecule is smaller than the unperturbed level splitting. In other words, all transition rates  $R_{if}^{L/R}$  of the FM system satisfy

$$R_{if}^{L/R}/n_B(\omega_i - \omega_f) \ll |\epsilon_i^0 - \epsilon_f^0|/\hbar, \quad (13)$$

where  $\epsilon_i^0, \epsilon_f^0$  are the unperturbed energies of the states  $|i\rangle, |f\rangle$ .

Another constraint is given by the fact that, near Breit-Wigner resonances, the current through the molecular magnet can be strongly increased due to coherent tunneling processes. This only holds at low temperatures, at higher temperatures the broadening of the thermal distribution destroys coherent tunneling, and the sequential

tunneling approach is valid again. The minimal temperature  $T$  for FM reservoirs is given by

$$R_{if}^{L/R}/n_B(\omega_i - \omega_f) \ll k_B T/\hbar. \quad (14)$$

All our calculations are at high enough temperature for the sequential tunneling approach to be valid for the FM system.

We note that relaxation of the state of the molecular magnet can be neglected as long as the coupling strength between the reservoirs and the molecular magnet exceeds the coupling between the molecular magnet and hyperfine- and phonon baths.

## VII. CONCLUSIONS

Using a sequential tunneling approach, we have studied transport of magnons and spinons through a triangular molecular magnet which is weakly coupled to two spin reservoirs. We have shown that, by changing the state of the molecular magnet through application of an electric- or magnetic field, we can control the magnitude of the spin current through the molecular magnet. We used this fact to propose a magnon-transistor, whose operation can be controlled by an electric- or magnetic field. We have shown for which parameters our transistor could operate at room temperature with a 11 ns switching time. We have shown how several magnon-transistors can be combined to create a NAND-gate.

## VIII. ACKNOWLEDGEMENTS

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